Localized vibrations in slider-block models

Gleb Morein and Donald L. Turcotte
Department of Geological Sciences, Snee Hall, Cornell University, Ithaca, New York 14853
(Received 6 November 1997)

We consider linear and two-dimensional arrays of slider blocks connected by springs. The blocks interact with a surface through a static coefficient of friction; the dynamic friction is taken to be zero so that energy is conserved. The initial energy of the system is prescribed and the only control parameter μ is proportional to the ratio of the mean initial energy to the static frictional force squared. ''Molecular-dynamics'' simulations of the temporal evolution of these systems have been carried out. For arbitrary initial conditions the arrays self-organize into patches of oscillating blocks with a dominant normal mode vibration. The fraction of blocks within the oscillating patches is almost independent of the prescribed energy, but the size of the patches systematically increases with increasing energy. The larger patches accommodate the larger available energies. The behavior of these systems is strikingly different from the behavior seen in regular slider-block models. [S1063-651X(98)10405-1]

PACS number(s): 05.40.+j, 02.70.Ns

I. INTRODUCTION

Burridge and Knopoff [1] introduced the coupled slider block model as an analog for earthquakes. The slider blocks are pulled over a surface by springs attached to a constant velocity driver plate and are also attached to each other by springs. If the static friction is greater than the slipping (dynamic) friction, stick-slip behavior is found. Huang and Turcotte [2] showed that two slider blocks exhibit classic chaotic behavior as long as there is any asymmetry in the system. In their study the equations of motion for the two blocks were solved simultaneously. A modification of this model is to allow only one block to slip at a time. The first block to become unstable is allowed to complete its harmonic motion before the stability of the second block is considered; if the second block is then unstable it is allowed to slip before the motion of the driver plate is updated. This then is a cellular-automata model. Extensive studies of this model were carried out by Narkounskaia and Turcotte [3] and its behavior was essentially identical to the results when both blocks were allowed to slip simultaneously.

Carlson and Langer [4] considered long linear arrays of slider blocks with each block connected by springs to the two neighboring blocks and to a constant-velocity driver. They used a velocity-weakening friction law and considered up to 400 blocks. Slip events involving large numbers of blocks were observed, the motion of all blocks involved in a slip event were coupled, and the applicable equations of motion had to be solved simultaneously. Although the system is completely deterministic, the behavior was apparently chaotic. Frequency-size statistics were obtained for slip events and the events fell into two groups: Smaller events obeyed a power-law (fractal) relationship, but there was an anomalously large number of large events that included all the slider blocks. The observed behavior was characteristic of self-organized criticality [5]. Nakanishi [6,7] studied multiple slider-block models using the cellular automata approach. A linear array of slider blocks was considered, but only one block was allowed to move in a slip event. The slip of one block could lead to the instability of either or both of the adjacent blocks, which would then be allowed to slip in a subsequent step or steps, until all blocks were again stable. Brown *et al.* [8] proposed a modification of this model involving a two-dimensional array of blocks. Many other studies of slider-block models have been carried out and these have been reviewed by Turcotte [9].

Rundle *et al.* [10] considered a multiple slider-block model in a mean-field approximation and showed that the energy fluctuations are characterized by Boltzmann statistics. Xu and Sornette [11] commented on this work and argued against thermalization. Morein *et al.* [12] considered a multiple slider-block model without a driver plate. The dynamic friction was set equal to zero so that energy was conserved and cellular-automata solutions were obtained. Only one block was allowed to slip during a time step. Sequential sweeps across the lattice were carried out. Again a Boltzmann distribution of energies was found. The purpose of this paper is to carry out a full dynamical simulation of this problem using a molecular-dynamics code.

II. MODEL

In this paper we consider one-dimensional linear arrays and two-dimensional square arrays of slider blocks; each block of mass m is connected to its neighbors with springs (spring constant k). There is no driver plate in this model. In both cases displacements are limited to the x direction. We assume zero dynamic friction and prescribe the static friction f_s between a block and the surface over which it is sliding. The initial total energy in the system is also prescribed. Because the dynamic friction is zero, the total energy in the system is preserved. Some blocks are unstable at a given time and are free to slip; the remaining blocks are stuck.

For the linear-array model a particular slider block is designated by the subscript i. The equation of motion of each block is

$$m\frac{d^2x_i}{dt^2} = f_i, \tag{1}$$

with the net force on the block f_i given by

$$f_i = k(x_{i-1} + x_{i+1} - 2x_i). \tag{2}$$

If a moving block reaches zero velocity and the net force on this block is less than the static friction threshold f_s the block will stick,

$$\frac{dx_i}{dt} \to 0, \quad f_i < f_s \text{ then } \frac{dx_i}{dt} = 0, \tag{3}$$

until the net force exceeds the static friction

$$f_i > f_s. \tag{4}$$

It is convenient to introduce the nondimensional variables

$$F_i = \frac{f_i}{f_s}, \quad X_i = \frac{x_i k}{f_s}, \quad T = t\left(\frac{k}{m}\right)^{1/2}, \quad V_i = \frac{\sqrt{km}v_i}{f_s},$$

$$E_s = \frac{k e_s}{f_s^2}. \tag{5}$$

The governing equations (1)–(3) become

$$\frac{d^2 X_i}{dT^2} = F_i \tag{6}$$

$$F_i = X_{i-1} + X_{i+1} - 2X_i \tag{7}$$

with the stick condition

$$\text{if} \quad \frac{dX_i}{dT} \to 0, \quad F_i < 1 \quad \text{then} \quad \frac{dX_i}{dT} = 0 \tag{8}$$

until the slip condition

$$F_i > 1. \tag{9}$$

At $T=0$ the blocks are given a random distribution of displacements; the resulting energy in the spring s is e_s. The mean energy in the springs at $T=0$ is $\overline{e_s}$. Since no energy is dissipated by dynamic friction (the dynamic friction is taken to be zero), energy is conserved and $\overline{e_s}$ is a constant independent of time. It is convenient to introduce the nondimensional energy parameter μ,

$$\mu = \frac{k\overline{e_s}}{f_s^2}. \tag{10}$$

This is the only parameter in the problem. If μ is large, very few of the blocks would be expected to stick; if μ is small, a large fraction of the blocks would be expected to stick. At any given time the system can be viewed as a collection of clusters of moving blocks with stuck blocks on the cluster boundaries. If the force on a boundary block of a cluster is greater than one, then that block will join the cluster. If during the motion of a block it has zero velocity and the force at this moment is less than one, the block will stick. If it is an edge block, the block will be removed from the cluster; if it is an interior block, the cluster will break into two clusters.

Our primary method of solution is a ''molecular-dynamics'' (MD) algorithm for the numerical integration of Eqs. (6)–(8) for all blocks simultaneously. At each time step the slip condition (9) for stuck blocks was checked. Whenever the velocity of an oscillating block went through zero the stick condition (8) was checked and if the force on this block was less than one the velocity was set to zero.

We have carried out a series of simulations using a MD algorithm on slider-block arrays with lengths up to 4096 blocks for one-dimensional (1D) and 64×64 for 2D arrays. Periodic and free boundary conditions were considered, giving basically the same results. A random distribution of initial displacements was given to the blocks corresponding to the specified value of the energy parameter μ. In each simulation a time-dependent transient was observed. After this transient a steady-state distribution of oscillating clusters was obtained. Different time steps τ with different ''Runge-Kutta'' schemes were tested to ensure no dependence on the time discretization. We found that the time needed to obtain a steady-state distribution and statistical properties of the system were independent of τ for sufficiently small τ.

Our simulations showed that independent of the dimension of the array (1D or 2D), boundary conditions (free or periodic), type of random initial conditions, and for a large range of the energy parameter μ the system self-organizes into a final steady state with the fraction of stuck blocks essentially constant independent of μ and a large fraction of the energy (generally more than 99%) localized in clusters of slipping blocks. The size of the clusters of slipping blocks increases systematically with increasing values of the energy parameter μ.

In the final steady state it is appropriate to analyze the distribution of harmonic modes in the clusters. For the 1D model the motion of the kth block in a cluster of size n is a combination of harmonic modes for the linear chain of blocks with fixed boundaries

$$X_k = \sum_{l=1}^{l=n} C_l \sin\left[\frac{lk\pi}{n+1}\right] \sin(\omega_l T + \alpha_l), \quad k,l \in [1,\ldots,n], \tag{11}$$

where the frequency ω_l of mode l is given by

$$\omega_l = 2\sin\left[\frac{l\pi}{2(n+1)}\right], \tag{12}$$

with corresponding amplitude C_l and phase α_l.

This modal expansion can also be used to formulate an alternative ''semianalytical'' algorithm for the time evolution of a linear array. Equations (11) and (12) completely specify the behavior of a cluster of n blocks. The lifetime of a cluster is defined to be time between when the cluster is created and the time when a moving block in the cluster sticks or a boundary block slips. In writing Eq. (11) we have assumed that the boundary blocks have zero displacement. This is not true, in general, so a shift and rescaling of the blocks coordinate X are required. However, as previously stated, the total energy in the stuck blocks in the steady state is less than 1%. The frequencies ω_l are unaffected by these boundary adjustments.

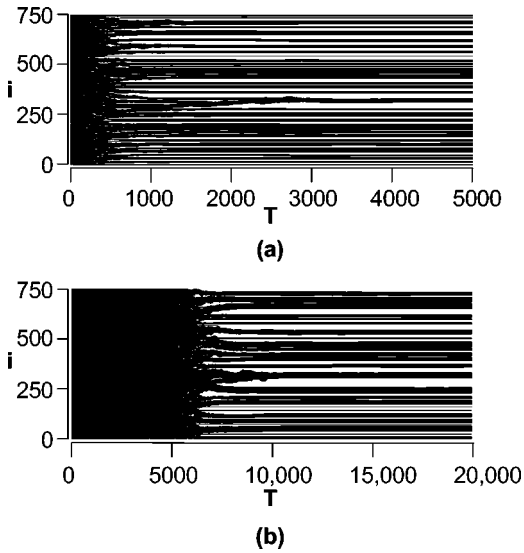

FIG. 1. Transient evolution of a linear model of 750 blocks. The array configuration is shown vertically with slipping blocks black and stuck blocks white. Time T is shown horizontally. A random distribution of energies (displacements) is given to the blocks initially ($T=0$). The development of well-defined clusters of slipping blocks is clearly illustrated. All quantities are dimensionless. (a) The energy parameter $\mu=2.0$ and each fifth time step is shown over a total time $T=5000$. (b) The energy parameter $\mu=8.0$ and each 20th time step is shown over a total time $T=20\,000$.

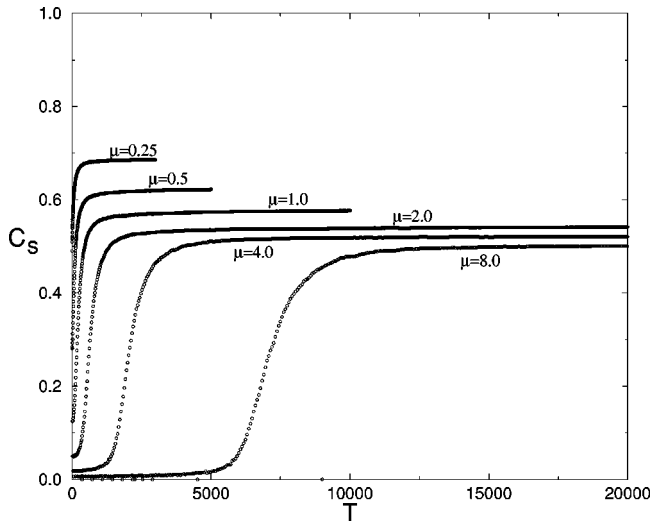

FIG. 2. Transient evolutions of linear arrays of 4096 blocks. The fraction of stuck blocks C_s is given as a function of time T for six values of μ. Each curve is obtained by averaging 25 simulations. All quantities are dimensionless.

The semianalytical method starts with arbitrary initial conditions. We identify clusters of moving blocks or blocks with zero velocity and a net force greater than one. We next determine the C_l, α_l, and ω_l from Eqs. (11) and (12). The third step is to find the shortest lifetime among the clusters. This is done by the numerical solution of the applicable set of algebraic transcendental equations and the stick and slip conditions from Eqs. (9) and (8). The cluster geometry is corrected and the sequence of steps is repeated until a steady-state solution is obtained. We compared this approach with the MD method for a linear array of four blocks and the results were virtually identical.

III. ONE-DIMENSIONAL SIMULATIONS

We first present the results of MD simulations for the time evolution of 1D arrays of slider blocks. The time evolution for two typical examples with 750 blocks is given in Fig. 1; fixed edge conditions were used. The array configuration is shown vertically with slipping blocks black and stuck blocks white. The nondimensional time T is shown horizontally. A random distribution of energies (displacements) is given to the blocks initially ($T=0$). The results for $\mu=2$ are shown in Fig. 1(a); each fifth time step is printed and the total nondimensional time is $T=5000$. Initially, relatively few blocks are stuck and the energy is randomly distributed. As time evolves clusters of slipping blocks form with the energy concentrated in these clusters; many more blocks stick. Eventually, a steady-state configuration is reached. The results for $\mu=8$ are shown in Fig. 1(b); each 20th time step is printed and the total nondimensional time is $T=20\,000$. The time evolution is similar to that for $\mu=2$ except the transient is longer and the clusters of slipping blocks are larger.

The transient behavior of the simulation is further illustrated in Fig. 2, where the fraction of stuck blocks C_s is given as a function of time T for several values of the energy parameter μ. It is seen that the length of the transient increases systematically with increasing values of μ. It is also seen that the fraction of stuck blocks C_s for large times has only a weak dependence on μ, particularly for large μ.

The transient behavior of the system is a process involving the merger and breakup of clusters of slipping blocks. For each value of μ there is a preferred size for the stable clusters that eventually emerge. The fraction of the blocks that are slipping is relatively constant; increasing the average energy increases the size of the clusters. The system self-organizes into a steady-state configuration in which the mean cluster size has a systematic dependence on the energy parameter μ.

The fraction of stuck blocks at equilibrium C_{sf} is given in Fig. 3 as a function of the energy parameter μ. The means and standard deviations are given for a large number of simulations. In the energy range $2<\mu<32$ the fraction of stuck blocks C_{sf} decreases only from 0.58 to 0.48. This further illustrates the relatively weak dependence of C_{sf} on μ for large μ.

To illustrate the behavior of the clusters, the steady-state distribution of energies E_i and velocities V_i in a linear array of 256 blocks with $\mu=1$ is given in Fig. 4(a) and with $\mu=8$ in Fig. 4(b). The energy of a block is taken to be the sum of the kinetic energy of the block and one-half of the potential energy in the two adjacent springs. The kinetic energies are taken at an arbitrary time T so that some clusters have relatively small velocities compared to the total energy for blocks in that cluster, which remains constant during the cluster lifetime (harmonic motion). The blocks appear to have a near sinusoidal standing-wave dependence of velocities within a cluster with the oscillations of adjacent blocks 180° out of phase. It is clear that there is a systematic increase in the maximum energy with cluster size. As the energy parameter μ is increased the size of the steady-state

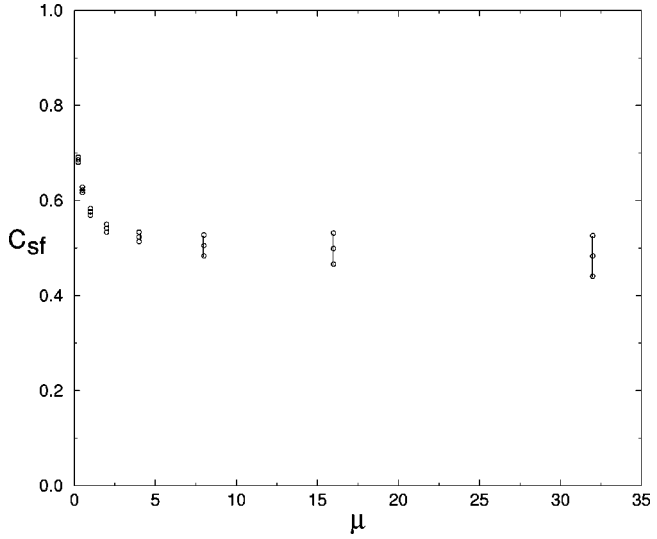

FIG. 3. Steady-state fraction of stuck blocks C_{sf} given as a function of the energy parameter μ. The mean and standard deviation are given at each energy for 25 simulations. All quantities are dimensionless.

clusters also increases, allowing the same number of slipping blocks to accommodate the larger amount of energy available.

In order to better understand the structure of the steady-state clusters we have carried out spectral analyses of the slip velocity of a block within a cluster that has reached the steady state. Typical examples for clusters with 22 and 32 oscillating blocks are given in Fig. 5. The spectral power density P of the velocity of an inner cluster block is given as a function of frequency ω. The excitation of the frequencies given in Eq. (12) is clearly illustrated. The highest frequency is always dominant with the others suppressed by an order of magnitude or more. All resonant frequencies with respect to the highest frequency are missing from the power spectrum. In the steady state the blocks in a cluster exhibit a well-defined harmonic oscillation. This harmonic oscillation evolves during the initial transient. At $T=0$ a random distribution of modes, a white noise, was introduced. The nonlinear static friction mixes these modes until a single harmonic becomes dominant in a cluster.

Our spectral studies show that the highest-frequency mode in Eq. (11) dominates. Thus we consider only the mode $l=n$ and Eqs. (11) and (12) become

$$X_k = C_n \sin\left[\frac{nk\pi}{n+1}\right] \sin(\omega_n T + \alpha_n), \tag{13}$$

$$\omega_n = 2\sin\left[\frac{n\pi}{2(n+1)}\right]. \tag{14}$$

In order for the blocks on the edge of a cluster to remain stuck the maximum nondimensional force on these blocks must be less than one from Eq. (8). We will assume that the actual force on these blocks is $F=1/2$ and will show that this is consistent with our results.

The maximum displacement of the edge block is obtained by taking $k=1$ and the $F=1/2$ force condition requires that

$$C_n = \frac{1}{2\sin\left[\frac{n\pi}{n+1}\right]} = \frac{1}{2\sin\left[\frac{\pi}{n+1}\right]}. \tag{15}$$

The motion of all n blocks in a cluster of size n is now specified.

We next find the amplitude of the motion of the central block in a cluster with an odd number of blocks, that is, $n=2m+1$ with $k=m+1$. From Eqs. (13) and (15) we have

$$X_{m+1} = \frac{\sin\left[\frac{\pi n}{2}\right]}{2\sin\left[\frac{\pi}{n+1}\right]} = \frac{(-1)^{(n-1)/2}}{2\sin\left[\frac{\pi}{n+1}\right]}. \tag{16}$$

The total energy of the central block in a cluster is the maximum kinetic energy E_{kem} of the block and is given by

$$E_{kem} = \frac{\omega_n^2 X_{m+1}^2}{2} = \frac{1}{8\sin^2\left[\frac{\pi}{2(n+1)}\right]}. \tag{17}$$

For very large clusters $(n\to\infty)$ this becomes

$$E_{kem} \to \frac{n^2}{2\pi^2}. \tag{18}$$

These results are compared with observations in Fig. 6. It is seen that there is good agreement.

The total energy in a cluster of size n, E_{cl}, can be obtained by summing the maximum kinetic energies of the blocks in the cluster. The result is

$$E_{cl} = \frac{n+1}{16\sin^2\left[\frac{\pi}{2(n+1)}\right]}, \tag{19}$$

which for very large clusters becomes

$$E_{cl} \to \frac{n^3}{4\pi^2}. \tag{20}$$

These results are compared with observations in Fig. 7. Again, good agreement is found.

The analysis given above shows that the energy in large clusters $(n\gg 1)$ scales with the size of the cluster according to $E_{cl}\sim n^3$. Thus the larger the energy the larger the mean size of the clusters required to contain the energy. Figures 1 and 2 show that the transient behavior results in a self-organization of clusters in which approximate normal mode oscillations accommodate the initial energy. The nonlinear stick-slip behavior mixes the initially random modes to the approximate normal mode behavior observed in the steady state.

Although there is a statistical distribution of cluster sizes, there is a systematic dependence of the mean cluster size $\bar{n}$ on energy μ. The distribution of cluster sizes $f(n)$ is given in Fig. 8 for four nondimensional energies $\mu=0.5$, 1.0, 2.0, and 4.0. Although there are relatively large numbers of clus-

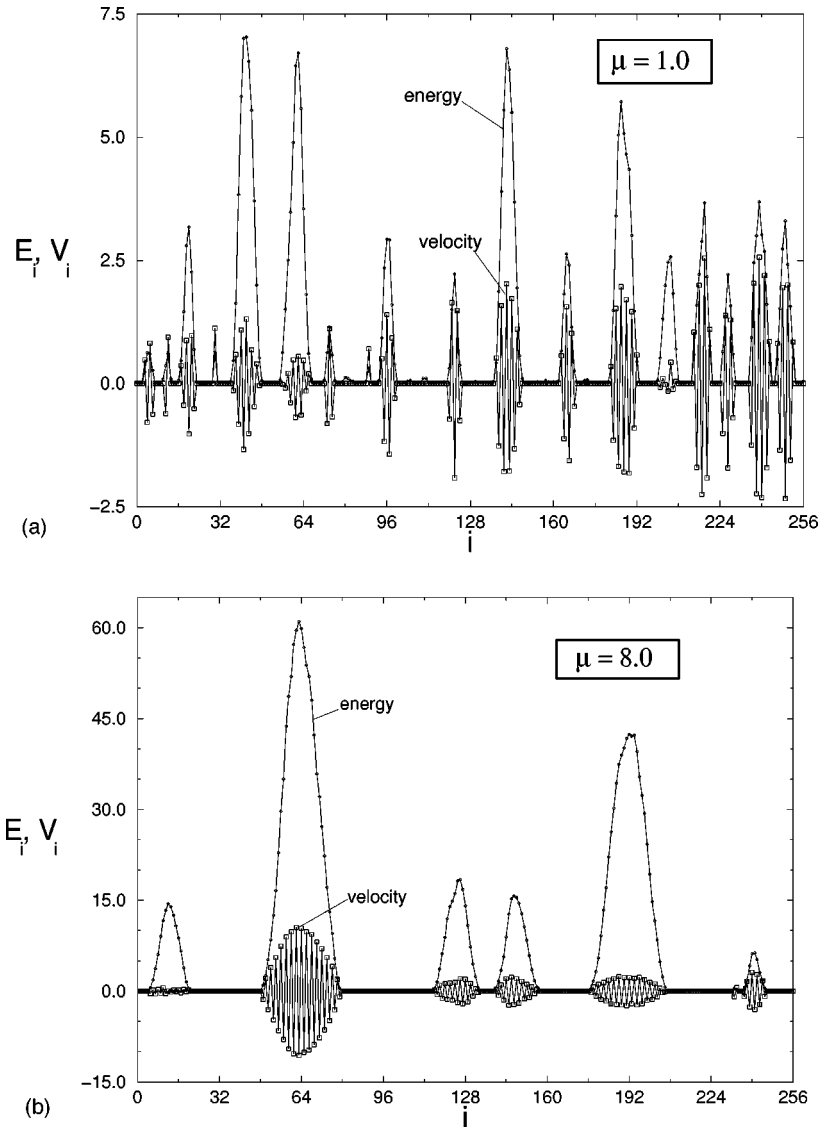

FIG. 4. Steady-state distributions of energies E_i and velocities V_i given for a linear array of 256 blocks with (a) $\mu = 1$ and (b) $\mu = 8$. All quantities are dimensionless.

ters with $n = 1,2$, these contain a very small fraction of the slipping blocks. There is a well-defined cluster size that contains the most blocks and this is seen to increase systematically with increasing μ. In order to quantify the dependence of cluster size on energy we consider n_{cl} clusters each with size $\overline{n}$. Again assuming that one-half of the N blocks are slipping, we can write

$$0.5N = n_{cl}\overline{n}. \tag{21}$$

The cluster energy E_{cl} is related to the energy parameter μ by

$$N\mu = n_{cl}E_{cl}. \tag{22}$$

Noting that $E_{cl} \sim \overline{n^3}$ from Eq. (20), we can combine Eqs. (21) and (22) to give

$$\overline{n} \sim \sqrt{\mu}. \tag{23}$$

The dependence of the mean cluster size from Fig. 8 on the energy parameter μ is given in Fig. 9. The best-fit straight line is a power law $\overline{n} \sim \mu^{0.53}$, which is in excellent agreement with Eq. (23).

IV. TWO-DIMENSIONAL SIMULATIONS

We have also carried out a series of MD simulations on 2D square slider-block arrays. The nondimensional variables introduced in Eq. (5) and the nondimensional energy introduced in Eq. (10) remain valid. A particular slider block is designated by subscripts i (position in the x direction) and j (position in the y direction). Displacements are restricted to the x direction, so that the governing nondimensional equations become

$$\frac{d^2X_{i,j}}{dT^2} = F_{i,j}, \tag{24}$$

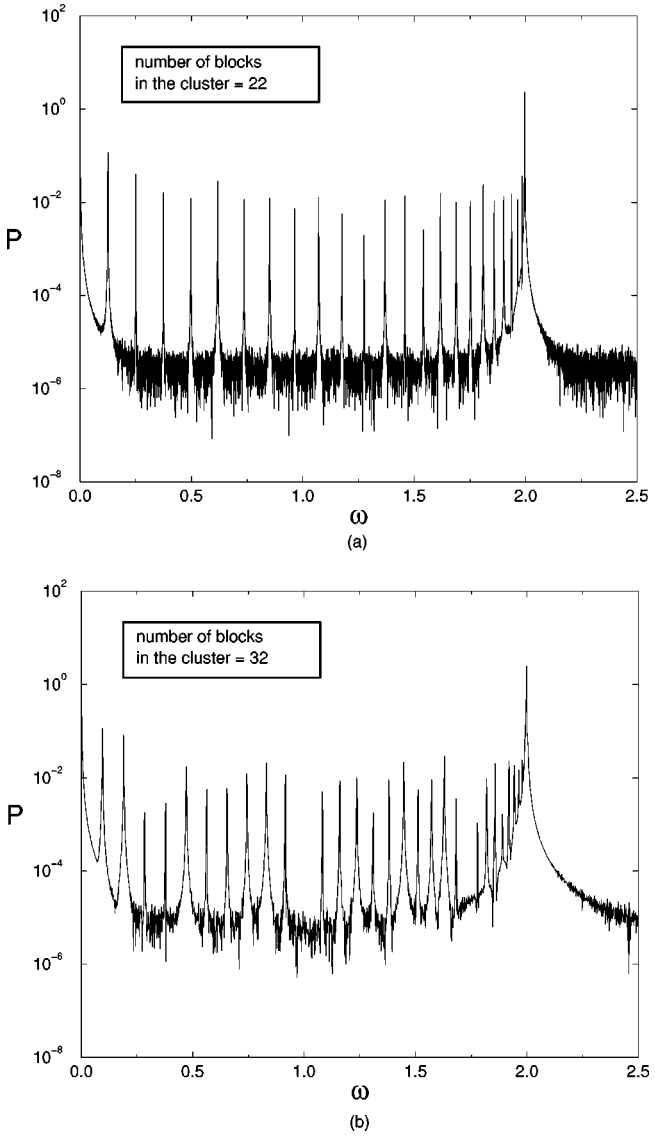

FIG. 5. Power spectral density P as a function of frequency ω for the velocity $V(T)$ of a slipping block in steady-state clusters. We consider $\mu=8.0$ and cluster size (a) $n=22$ and (b) $n=32$. All quantities are dimensionless.

$$F_{i,j}=X_{i-1,j}+X_{i+1,j}+X_{i,j-1}+X_{i,j+1}-4X_{i,j}, \qquad (25)$$

with the stick condition

$$\text{if} \quad \frac{dX_{i,j}}{dT}\to 0, \quad F_{i,j}<1 \quad \text{then} \quad \frac{dX_{i,j}}{dT}=0 \qquad (26)$$

until the slip condition

$$F_{i,j}>1 \qquad (27)$$

instead of Eqs. (6)–(8).

The time evolution for a 64×64 array with $\mu=2.5$ is given in Fig. 10. The slipping blocks are white and the stuck blocks are black. Under the random initial conditions given at $T=0$ few blocks are stuck. Soon patches of slipping blocks form and these patches migrate, change shape, collide, and break up. Eventually, a steady state is established with nearly circular patches with a normal mode behavior.

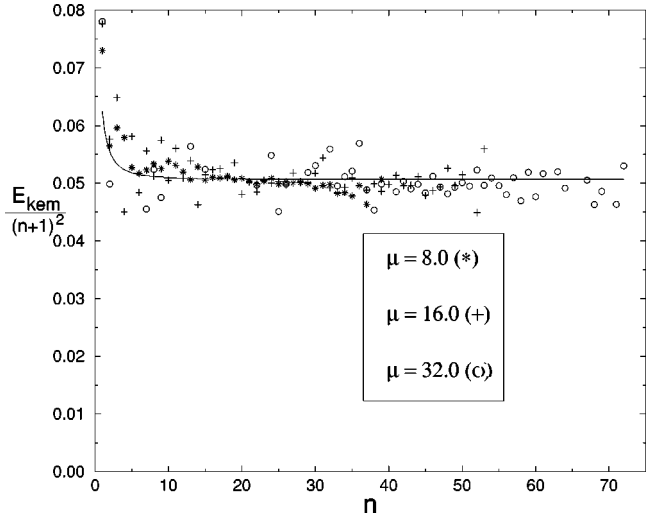

FIG. 6. Dependence of the maximum energy in a cluster E_{kem} divided by $(n+1)^2$ on cluster size n given for energy parameters $\mu=8.0$, 16.0, and 32.0. The results are in good agreement with the prediction given in Eq. (17). All quantities are dimensionless.

The transient sticking and slipping of blocks has converted the initial random phases and amplitudes (white noise) to a series of steady-state, drum-head-like normal mode oscillations.

The transient behavior is further illustrated in Fig. 11, where the fraction of stuck blocks C_s is given as a function of time for several values of μ. Once again the transient self-organization leads to a fraction of stuck block being approximately constant $C_s\sim 0.76$ over a relatively wide range of energies.

In Fig. 12 the final equilibrium configurations are given for several different energies. In each case nearly circular patches form with normal mode behavior. The size of the patches increases systematically with increasing energy as in the 1D simulations. The general behavior of the 2D simulation is very similar to the 1D simulations discussed above.

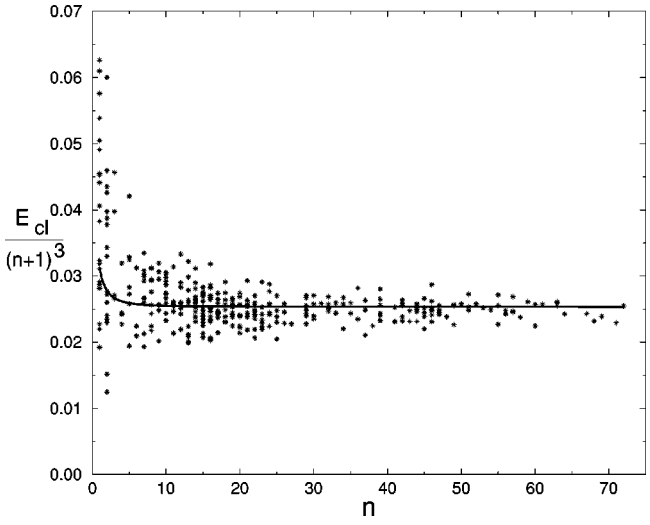

FIG. 7. Dependence of the cluster energy E_{cl} divided by $(n+1)^3$ on cluster size n given for energy parameters $\mu=4.0$, 8.0, 16.0, and 32.0. The results are again in good agreement with the prediction given in Eq. (19). All quantities are dimensionless.

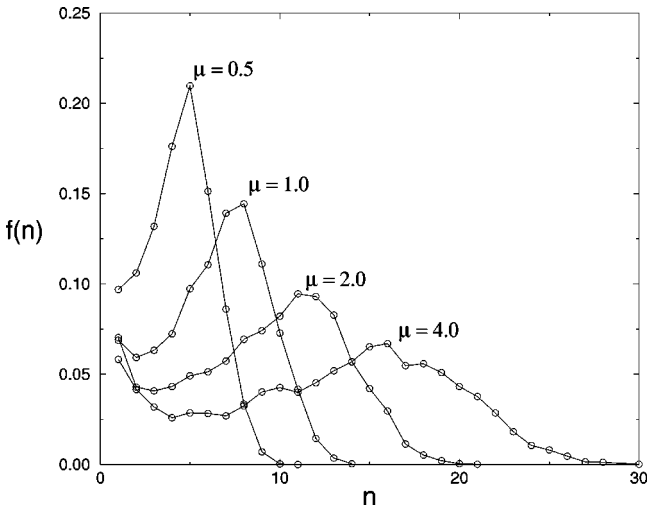

FIG. 8. Distribution of cluster sizes $f(n)$ given as a function of n for energy parameters $\mu=0.5$, 1.0, 2.0, and 4.0. These results are for a linear array of 4096 blocks and 25 realizations have been averaged. All quantities are dimensionless.

V. DISCUSSION

We have considered arrays of slider blocks connected by springs that exhibit stick-slip behavior due to a frictional interaction with a surface. The dynamic friction is taken to be zero so that energy is conserved. The governing equations are linear except for the stick-slip behavior; this behavior introduces a strong nonlinearity.

A series of numerical simulations have been carried out on both linear arrays of blocks and square, two-dimensional arrays of blocks using molecular-dynamic algorithms. Similar evolutionary behaviors are found in the two cases. The only parameter in the problem is the energy parameter μ. For large values of μ the mean energy is large compared to the static friction and a large fraction of the blocks would be expected to slip. For small values of μ the mean energy is small and a small fraction of the blocks would be expected to slip. However, this is not the behavior observed as an array approaches a steady state.

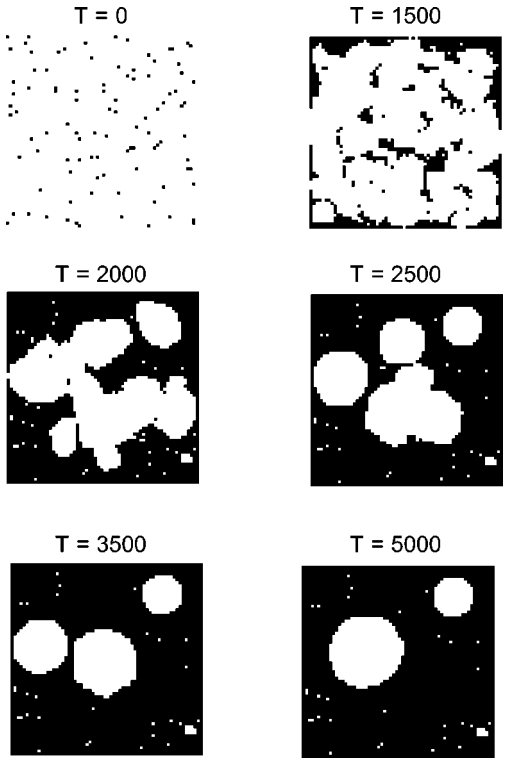

FIG. 10. Transient evolution of a 64×64 square array of slider blocks with energy parameter $\mu=2.5$. The slipping blocks are white and the stuck blocks are black. The evolution of the system is shown at six times. The development of nearly circular clusters of slipping blocks is clearly illustrated. All quantities are dimensionless.

Initially, we start the arrays with a random distribution of energies in the blocks that corresponds closely to a white noise or random distribution of normal modes. In both the 1D and 2D arrays a transient period of self-organization oc-

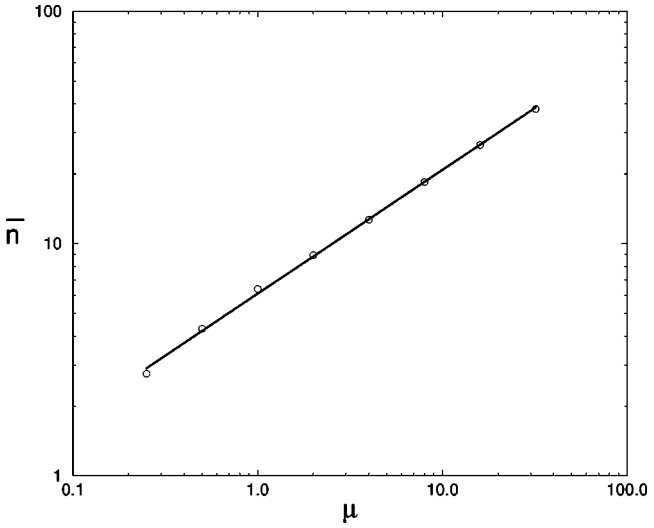

FIG. 9. Dependence of the mean cluster size $\bar{n}$ from Fig. 8 on the energy parameter μ. The best-fit straight line has a slope of 0.53 compared to the value 0.5 from Eq. (23). All quantities are dimensionless.

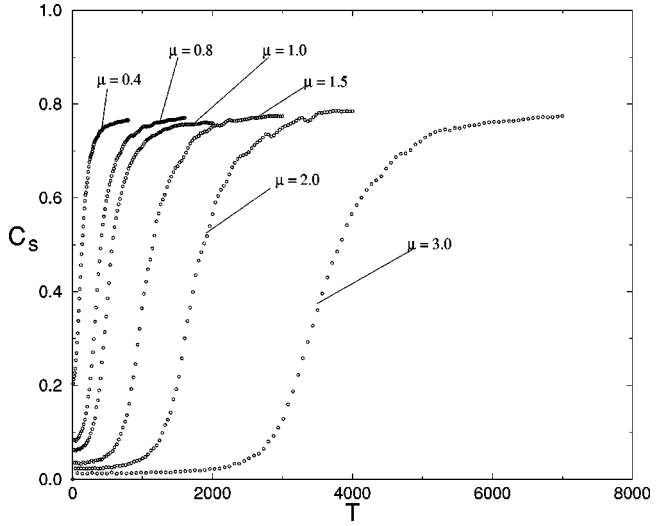

FIG. 11. Transient evolution of two-dimensional arrays of 64 $\times$64 blocks. The fraction of stuck blocks C_s is given as a function of time T for the values $\mu=0.4$, 0.8, 1.0, 1.5, 2.0, 3.0, and 4.0. All quantities are dimensionless.

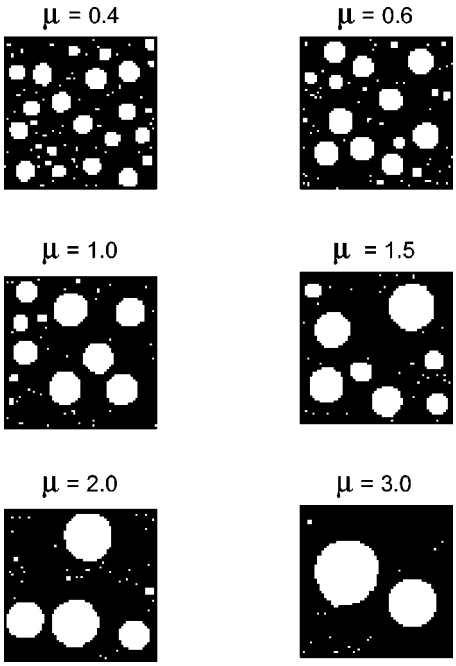

FIG. 12. Steady-state configurations of the two-dimensional system of size 64×64 given for energy parameters $\mu=0.4$, 0.6, 1.0, 1.5, 2.0, and 3.0. All quantities are dimensionless.

curs in which clusters of slipping blocks evolve. Eventually a steady state is reached without further evolution. In this steady state the slipping clusters exhibit a normal mode behavior with the harmonic motion of adjacent blocks in a cluster being 180° out of phase and the energies of blocks having a sinusoidal dependence on position across the cluster. The 2D clusters have nearly circular planforms and have drum-headlike normal mode oscillations.

In both one and two dimensions the fraction of stuck blocks is nearly independent of energy over quite a wide range of energies. However, the size of the clusters has a systematic dependence on energy, with the mean cluster size increasing systematically with energy. The energy in the edge blocks in a cluster is prescribed as a fraction (~0.5) of the maximum force a stuck block can sustain. In one dimension the energy of a cluster E_{cl} scales with the size n_c of a cluster according to $E_{cl}\sim n_c^3$. Thus larger clusters can accommodate more energy. The nonlinear stick-slip behavior converts and self-organizes the initial random mode behavior to a nearly normal mode behavior.

We observe in both one and two dimensions that the fraction of stuck blocks is essentially independent of the energy for relatively large energies ($\mu>1$). This can be explained qualitatively in terms of a scale-invariant cascade. Small clusters collide forming larger clusters until the clusters are large enough to absorb the available energy. This collision process is independent of the size of the clusters colliding so that the fraction of stuck blocks is also scale invariant and therefore constant.

It is interesting to compare the behavior of our slider-block model with the observation of oscillons in vertically vibrating granular layers [13]. An oscillon, in this context, is an isolated heap of grains that oscillates up and down. Because the oscillon is subharmonic, peaks and craters can coexist. They are also very stable lasting for more than 10^5 periods. In our model excitations are also localized and become time independent.

It is also of interest to point out the profound differences between a cellular automata solution to this problem [12] and the molecular-dynamics formulation presented in this paper. In the cellular automata formulation only one block was allowed to slip at a time; the remaining blocks were stationary. If the block considered was unstable it was allowed to complete a single harmonic oscillation before the next block was studied using a checker-board selection of blocks. In this case the energy distribution in the blocks thermalized and a modified Maxwell-Boltzmann distribution of energies was obtained.

In the molecular-dynamics formulation of the problem the equations of motion of slipping blocks are solved simultaneously and normal mode oscillations are obtained. Patches of slipping blocks evolve into an equilibrium structure in which the fraction of stuck blocks has only a weak dependence on energy, but the cluster size systematically increases with energy. In this formulation the self-organized critical behavior and chaotic behavior associated with driven, dissipative slider-block models is not observed.

In addition to the simple model presented here we have also found that the same self-organization process can be observed in more complex slider-block systems. In the presence of a stationary driver plate the system possesses a similar set of fundamental harmonic modes. With a slow moving driver plate clusters of moving oscillating blocks can follow the driver plate. Fundamental modes in this case were found to be a disturbed version of harmonic modes for the system without driving. There was no stationary state and the lifetimes of the clusters were finite. In addition, a small dynamic friction can be accommodated. We have not studied these cases in detail as molecular-dynamic simulations become very time consuming.

We have also found it interesting to compare our results with models used to describe localized vibrations in perfect anharmonic crystals [14,15]. These physics models demonstrate that self-localized vibrational modes can be stable for many types of interatomic potentials. The mechanics of localization is somewhat different as the frequency of these localized modes is above the spectrum found for the linear approximation of the interatomic potential.

ACKNOWLEDGMENTS

It is a pleasure to thank Andrei Gabrielov, Sergei Kiselev, Jim Sethna, John Rundle, and Bill Newman for useful discussions and comments. This research was partially supported by NSF Grant No. EAR 94-23818.

[1]

[1] R. Burridge and L. Knopoff, Seis. Soc. Am. Bull. **57**, 341 (1967).
[2] J. Huang and D.L. Turcotte, Geophys. Res. Lett. **17**, 223 (1990).
[3] G. Narkounskaia and D.L. Turcotte, Geophys. J. Int. **111**, 250 (1992).
[4] J.M. Carlson and J.S. Langer, Phys. Rev. A **40**, 6470 (1989).
[5] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. A **38**, 364 (1988).
[6] H. Nakanishi, Phys. Rev. A **41**, 7086 (1990).
[7] N. Nakanishi, Phys. Rev. A **43**, 6613 (1991).
[8] S.R. Brown, C.H. Scholz, and J.B. Rundle, Geophys. Res. Lett. **18**, 215 (1991).
[9] D.L. Turcotte, *Fractals and Chaos in Geology and Geophysics*, 2nd ed. (Cambridge University Press, Cambridge, 1997).
[10] J.B. Rundle, W. Klein, S. Gross, and D.L. Turcotte, Phys. Rev. Lett. **75**, 1658 (1996).
[11] H.J. Xu and D. Sornette, Phys. Rev. Lett. **78**, 3797 (1997).
[12] G. Morein, D.L. Turcotte, and A. Gabrielov, Geophys. J. Int. **131**, 552 (1997).
[13] P.B. Umbanhowar, F. Melo, and H.L. Swinney, Nature (London) **382**, 793 (1996).
[14] S.A. Kiselev, S.R. Bickham, and A.J. Sievers, Comments Condens. Matter Phys. **17**, 135 (1995).
[15] S.R. Bickham, S.A. Kiselev, and A.J. Sievers, Phys. Rev. B **47**, 14 206 (1993).